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IMAGERY AND PROPERTY NOTICING: YOUNG STUDENTS' PERCEPTIONS OF THREE-DIMENSIONAL SHAPES

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A study of how students attempt three-dimensional problems presented as diagrams illustrates the range of thinking used by middle primary school students. The study involved giving items to 265 students and interviewing six additional students. The spatial ability of reseeing assists students to notice properties of shapes held in their imagery. The verbalized knowledge associated with noticing aspects of their imagery and changes in shape and orientation are developing prior to a consistent description of the three-dimensional shapes. This finding has implications for teachers.

INTRODUCTION

This paper looks at how young children may attempt items about three-dimensional shapes (3D shapes or solids) presented diagrammatically. The literature on spatial abilities is vast and identifies different kinds of spatial abilities from different paradigms and with different nomenclatures (Eliot, 1987). One spatial ability requires students to see a shape or part of a shape, disembedding it from the configuration in which it is presented. For example, some young children find it hard to disembed or see an angle on a shape while older students may find it difficult to see a triangle formed by a diagonal and adjacent sides of a parallelogram when both diagonals are drawn (see Figure 1). Disembedding is a different kind of skill from those requiring mental manipulation of images or viewing from another perspective (Eliot, 1987; Tartre, 1990).

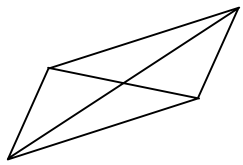

Figure 1: Disembedding a triangle formed by adjacent sides and a diagonal

The reverse skill to disembedding is that of completing figures or fitting parts together (also called integration of detail by Pellegrino & Hunt, 1991 and spatial relations by Johnston & Meade, 1985). Tartre (1990) referred to re-seeing for both two-dimensional (2D) and three-dimensional (3D) shapes constituting both disembedding and embedding skills.

The discipline of noticing has been linked to attention in several theoretical studies (Owens, 1998 & Mason, 2003). Both emphasise the importance of the person's existing imagery and concepts as well as the current context for learning and what focuses attention. In Owens' (1998) study, participants who listened to a voice directing their attention had significantly less variance in approaches to a visually presented problem compared to the variance in approaches of participants without this auditory information. Mason (2003) illustrated from experiences that "attention shifts back and forth from whole to part, and part to whole, and from part as part to part as whole and so to parts of parts: from wholeness to discriminating parts through stressing and ignoring." Owens (1998) explained the importance of the link between visual imagery and conceptual knowledge in the process of attention. Attention is internal and can be influenced by thinking about what is being observed and abstracting relationships from the specifics (Bennett, 1993 cited in Mason, 2003).

Several classroom studies have shown how important this skill of disembedding is in students' concept development. Owens (1996) illustrated how some Year 2 children developed the concept of an angle on a shape by first identifying the arms of the angle using thumb and forefinger. The children then associated the words big, small and middle-sized angles to the different angles on tangram puzzle pieces. Ambrose and Falkner's classroom study (2002) shows that children have difficulty in noticing edges of polyhedra (3D shapes with flat faces). They pointed out the need for students to talk about the properties of the polyhedra that they made. They illustrated how young children could identify polyhedra within a structure made of several polyhedra but they did not necessarily identify the adjoining edges.

Re-seeing is an important skill for Pirie and Kieran's (1992) schema for learning at the "property noticing" phase. Before this phase, Pirie and Kieran describe learning as moving from "primitive knowing" to "image making" and then to the phase of "image having". Imagistic knowledge can be acted upon by noticing properties. Martin and Pirie (2003) provide details of each of these phases of learning and emphasise how important having an image is for students to move from property noticing to fold back and rethink what they are perceiving in the image. In this process of re-seeing, the students develop concepts by noticing properties in different ways. Clements, Swaminathan, Hannibal and Sarama (1999) suggest property noticing begins with a synthesis of verbal and imagistic knowledge. It is this theoretical perspective that became evident when analysing the data in the current study.

Students have been shown to use analysis as well as mental rotation for the same items and a plethora of skills have been discussed in information processing studies leading Lohman, Pellegrino, Alderton and Regian, (1987) to state in basic terms that

> Visual stimuli must be held in sensory memory while encoding processes (or pattern matching productions) operate to identify all or parts of the stimuli …Spatial ability may not consist so much in the ability to transform an image as in the ability to create the type of abstract, relation-preserving structure on which these sorts of transformations may be most easily and successfully performed (pp. 273-274).

Elia, Gagatsis and Kyriakides (2003) used the notion of dynamic intuition to explain that younger children might understand shapes in a dynamic visual way so that lines and shapes are dynamically related. Changes to a shape may be quite dynamic so a square can be extended to be a rectangle in one dimension rather than being extended in two directions into another square. Similarly an equilateral triangle becomes an isosceles triangle when changed in one dimension only. Nevertheless Elia et al. (2003) found that younger children were more likely to use two dimensions than those who had been to school and learnt about the shapes in a non-dynamic, static way. They concurred with Van de Sandt (2001) that teaching should build on student's current (dynamic) approach to geometric thinking rather than follow a teacher's static thinking and classification approach. Kyriakides (1999, cited in Elia et al., 2003), Oberdorf and Taylor-Cox (1999), and Owens (1992) suggest teaching the general case (e.g. rectangle) and relative properties of shapes to develop the specific case (e.g. square) may be best achieved with dynamic procedures like stretching a square or pulling a point of a shape to make other shapes (e.g. as in computer drawing packages or with thin elastic loops, Owens, 2001b). Owens (2001a) and Clements et al. (1999) emphasise the disadvantage that a limited range of structured materials in schools can have on students' imagistic knowledge of shapes. Clements et al. (1999) suggested that a decrease in students' correct selection of figures for a category as they increased in age may be due to an emphasis on an irrelevant attribute of the limited prototypical image. When teachers over-emphasise limited properties such as a square has four equal sides and four corners or when learning experiences encourage limited visual skills, younger children may ignore important identifying properties of a shape (Smith, 1989) such as the perpendicularity of sides for identifying squares.

Another matter to consider in paper-and-pencil testing is that the skill of Interpreting Figural Representations may influence responses to items supposedly testing the skill of visual processing (Bishop, 1983). Johnstone and Meade (1985) developed group tests for primary aged students based on the Thurstone and Thurstone (1941) spatial abilities test. In order to improve the young children's understandings of the items, Johnstone and Meade introduced their tests by explaining the vocabulary with real objects such as blocks and cut-out shapes. Independently, I had decided to provide a similar introduction to my items in earlier versions of space tests (Owens, 1992) to take account of interpreting figural representation. Both Johnstone and Meade and myself included three-dimensional items. Experiments and testing of students with little experience of 2D representation of 3D indicated that such students can relatively quickly overcome the problem of interpreting figural representations (Lean, circa 1983).

THE STUDY

Most of the studies on visual imagery and shapes with young children use drawings relate to two-dimensional shapes. The study reported in this paper considered how students could recognise parts of 3D shapes, mentally join shapes to make a rectangular prism or other 3D shape, and decide whether a certain 3D shape could be tessellated to form a given shape. Many items were influenced by the examples of spatial abilities given by Eliot and McFarlane-Smith (1983) and so are heavily based on the psychological literature of spatial abilities in intelligence.

Participants in this study were 265 children in nine classes in three NSW state schools. Two schools were in south-western suburban Sydney and one in the countryside fringing this part of the city. One class was a Year 6 class and the other eight classes had children from Years 3 to 5 (mostly with composite grades). The test was not limited by time. The items discussed in this paper were part of a larger number of items on 3D spatial thinking (Owens, 2001a) which were Rasch analysed (Andrich, 1988). The Rasch analysis provides a validity measure of whether the responses for a particular item fit well with responses from other items by comparing the expected outcomes of ability subgroups of students as determined by all items with the responses on this particular item using a chi-squared analysis. A Cronbach Alpha reliability measure for the test was also obtained. The Rasch analysis provides a difficulty level for each item compared to other items. For each item, the percentage of students correct in each of the nine classes was also calculated.

The validity of the items was also checked by means of interviews with students to find out how they were thinking as they completed the items. It was not possible to interview students in the main study as they completed the items as a class group following the oral and concrete material introduction. A school neighbouring one of the Sydney suburban schools in the main study and considered similar in size and socio-economic and cultural background was selected. Six Year 4 students were interviewed immediately after completing each set of items to see how they described their thinking while carrying out each task. These students came from across the ability range as decided by the teacher based on class formative assessment and tests. These interviews inform us of what the young students noticed and how they were thinking while solving the items about blocks represented as drawings.

RESULTS

Overall there was high internal consistency between all the 51 items of the test given to the students (Cronbach Alpha = 0.90, Owens, 2001a). From the Rasch analysis, item difficulties ranged from -2 for easy items to 2 for hard items. When coupled with the range of percentages of students correct in any one class, it is possible to determine which items most students can do and which require more advanced mathematical thinking. In general, but not totally, the lowest percentages came from classes with Year 3 students and the highest from the Year 6 class. The selection of items are clustered under the type of spatial thinking required to complete the set of items. These are:

- recognizing shapes within shapes (disembedding)
- completing shapes by joining shapes together (embedding)
- tessellating blocks

Tables 1 to 4 indicate that the responses to specific items are quite variable for different aged classes.

Recognising Shapes within Shapes

Items that require students to recognize shapes within configurations of shapes are commonly called hidden figures. They require the student to select the 2D figure that incorporates a given 2D figure. The items here require recognition of partially hidden 3D shapes (Figure 2).

Part B2 Under Smiley is a building of 3 blocks. Is the 3D shape one of the blocks?

Circle Yes or No.

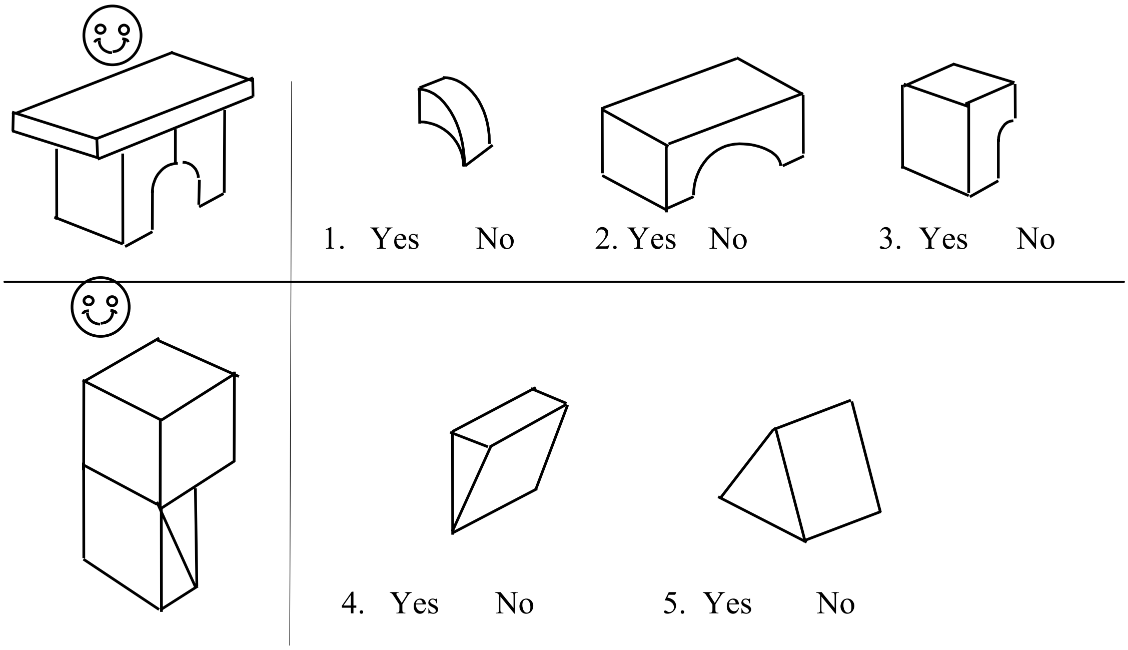

Figure 2: Recognising 3D shapes within shapes.

From Table 1, Item B2,1 illustrated in Figure 2, was marginal in fitting with other items on the Rasch analysis. The students gave interesting explanations and it seems they were settling down to this type of question. Those who thought the block was in the building of blocks, attended to the curve and three-dimensionality of the blocks. However, some degree of sophistication was needed in interpreting the figures as Bishop (1983) suggested. This was evident for Item B2,2 in which students needed to place some meaning on the vertical middle line of the block configuration. It may be for this reason that Item B2,2 on the Rasch analysis showed the lowest achieving group scored lower than expected. Students needed to have the visual skills of noticing and disembedding this line from the configuration and to disembed the individual blocks. They also needed to recognise the height differences and place meaning on the height which is a cognitive understanding. Smith (1989) suggested that it was common with younger students to not notice height differences.

Students did not hesitate in deciding the triangular shape block in B2,4 was embedded in the figure although only partially seen or turned. The general size of the block in B2,5 was the determining factor in not selecting this block and it was readily distinguished from the cube. Hence it was a very easy item.

	B2,1	B2,2	B2,3	B2,4	B2,5
Item Difficulty	-1.09*	0.97*†	0.54	-1.7	-2.06
Lowest Percentage Correct for a Class	90	60	79	86	89
Highest Percentage Correct for a Class	100	91	100	100	100

Note. * Scores for the item did not fit the spread of scores expected from the scores on other items. *† indicates that only one extreme group was more extreme and so fit was satisfied.

Table 1: Item difficulty and class percentages of students correct for items on seeing 3D shapes within shapes.

The following transcript illustrates that the student, Hayden, understood the diagram as representing 3D shapes and he pointed out what he disembedded and noticed in the configuration. He explains why these aspects of the diagram are important in his decision making.

Hayden:	That (points to B2,2) is not in the picture but that (points to shape under Smiley) is showing a line. It is like (B2,2) that but it doesn't have corners. And this one (points to B2,3) only has 2 corners but some at back and that (points to B2,1) joins at the end.
Interviewer:	This has corners but where are corners over here (under Smiley)?
Hayden:	There (pointing to parts below flat block).
Hayden:	4 (He moves on to the next set of diagrams in Part 2B and points to Item B2,4) That goes there (points to bottom section of arrangement under Smiley) and that (points to B2,5) isn't actually in the picture but for number 4 it was there oh, (unsure).
Interviewer:	Which part were you looking at?
Hayden:	Bottom, it couldn't be.

Hayden was relying on analysing the shapes in detail, referring back to the pictures several times illustrating that problems are being analysed rather than images rotated in making decisions (Lohman et al., 1987). However, we can see in this quote how Hayden was applying his current knowledge of reading diagrams, mostly of two dimensions, and some early play with blocks, to his current decision making. Nevertheless his responses remain relatively unsophisticated as he has a lack of verbal knowledge and of diagram analysis skills to be sure of his responses. Like the younger students in Clements et al.'s (1999) study on the simpler 2D shapes, Hayden is only just beginning to make use of properties to make decisions about the 3D shapes.

These items may be influenced by both the opportunity to play with blocks or wooden off-cuts and the opportunity to read these kinds of diagrams. Two of the interviewed students referred to playing with blocks recently in class.

Completing Shapes by Joining Shapes Together

In these items students needed to complete rectangular prisms or make new shapes. These tasks are also called spatial relations or embedding tasks.

Making Rectangular Prisms. Being able to mentally imagine putting two 3D shapes together to get a new shape may seem to be beyond the grasp of young children but results on these items indicate otherwise. Having discussed the term rectangular prism with many concrete examples during the introduction to the test, and the idea of joining two blocks together so they form a rectangular prism, the students were able to explain how the blocks could be turned and how they could be fitted together. Nevertheless, some items were more difficult to imagine fitting together than others. This is apparent for item B3,4 (Figure 3) where the block needs to be rotated and for which the lower aged classes had a low percentage of students correct (Table 2).

	B3,1	**B3,2**	**B3,3**	**B3,4**
Item Difficulty	-1.4	-0.61	-0.23	0.43
Lowest Percentage Correct for a Class	89	79	75	64
Highest Percentage Correct for a Class	100	100	94	94

Table 2: Item difficulty and class percentages of students correct for items on completing rectangular prisms.

Part B3 A 3D shape can be joined to the 3D shape under Smiley.

Can the shapes make a rectangular prism?

Circle Yes or No.

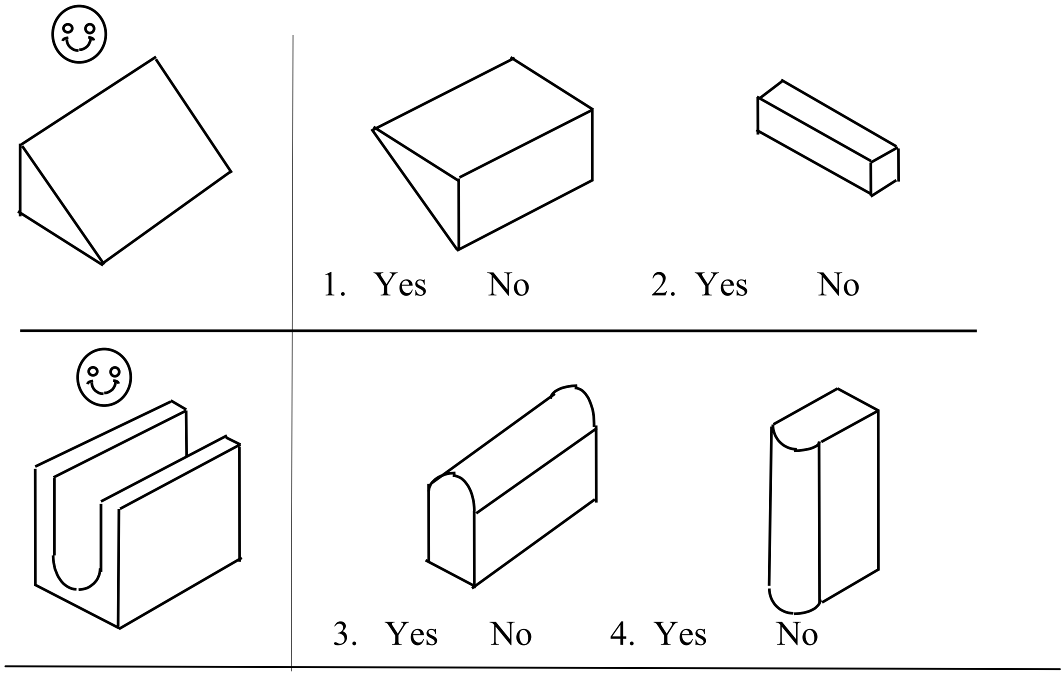

Figure 3: Joining two 3D shapes together to make a third shape.

Ahmed and Daniel changed their minds for B3,3 or B3,4 during the interview. Ahmed talked of putting in the piece to make it smooth and Daniel talked of “pushing it in, sliding it in and, if it stuck, hammering it in”. This comment would suggest some experience with fitting blocks together but perhaps none with curved sections. These are interesting examples of how students move from naïve primitive knowledge of the problem to more advanced focussing on features and relevant properties (Pirie & Kieran, 1992). They also illustrate the dynamic movement of objects in imagery. Hayden had a similar description for Items B3,3 and B3,4.

Hayden: That (B3,3) can go there (pointing to figure under smiley); turn it upside down. That (B3,4) turn over (indicates quarter turn with hand) and it can fit there.

In a rectangular prism you'd have a rectangle up the top here?

Interviewer: Where is the rectangle?

Hayden: The bottom of the end of these. That would go there and it would make a flat surface.

Again we see the synthesis of verbal and visual knowledge with the verbal descriptions being somewhat inadequate for clarity of the properties of the shape.

Michael was clear that B3,2 could not join with the block under Smiley. It would be "like a seesaw" and would not sit firmly. Rectangular prisms were seen as stable. However, some students did say “Yes” for B3,2 because it was a rectangular prism and rectangular prisms can be joined to make rectangular prisms but this was not being asked in this item. Perhaps this explains why the item was slightly harder than B3,1.

In summary, the students considered the flat and in some cases rectangular surface that would be made by joining the pieces together. Others noted how the curves would sit together. Again the percentages correct for the classes fluctuated. The students could spatially reason by mentally turning the blocks, analysing the shapes, and referring to their knowledge of the flat surfaces of rectangular prisms.

Joining 3D Shapes to Make New 3D Shapes. In Part C1, students needed to picture the result of joining two 3D shapes together. In general these items were well done. The blocks were visible and the items were done by analysis as well as mental rotation.

	C1,1	C1,2	C1,3	C1,4
Item Difficulty	-0.61	-1.25	-1.26	-1.55
Lowest Percentage Correct for a Class	83	90	90	90
Highest Percentage Correct for a Class	97	100	100	100

Table 3: Item difficulty and class percentages of students correct for items on recognising the blocks to make a 3D shape.

Part C1 Under Smiley are two blocks. Can they be joined to make the shape.

Circle Yes or No.

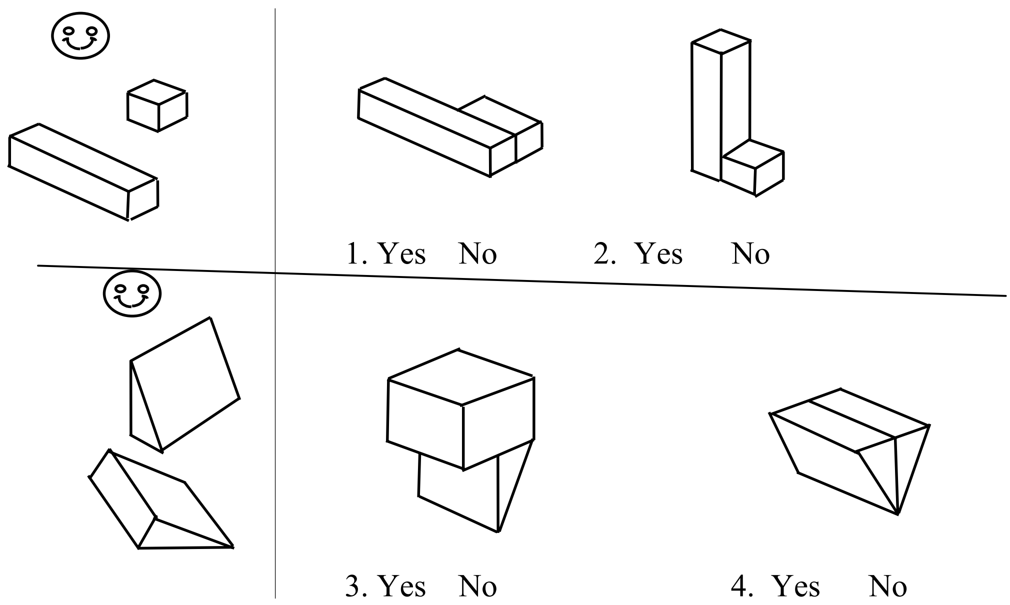

Figure 4: Recognising the joining of two 3D shapes to form another 3D shape.

Ian explained the items thus:

Ian: Because got long piece but don't have that (points to second block on C1,1). (Points to shapes under Smiley.) That is a square and that is a triangle [sic, means rectangle] so won't make it long enough for that (points to second block on C1,1). (Refers to C1,2) make that stand up and put little one next to it.

(Points to C1,3.) There is a square here and there is two triangles here (under Smiley).

Interviewer: Any more about shapes besides thinking about triangle?

Ian: If put that triangle same as that but facing other way, make a flat surface just like that (C1,4).

At this stage, the student is referring to key faces of the 3D shapes. He is also aware that two blocks will join together to give a flat surface. The decision seems to be more closely linked to experience with whole blocks than to experience with joining triangles per se. Other students also expressed similar approaches.

Tessellating Blocks

Students need to picture the tessellation of blocks for filling a 3D shape for understanding volume. Students generally found it easy to select whether blocks of the type illustrated will tessellate to form the required shape but they are less able to visualise the number of blocks. Hence students gave the incorrect number of blocks needed (see Table 4, answers labelled b).

	C2,1a	C2,1b	C2,2	C2,3	C2,4a	C2,4b	C2,5a	C2,5b
Item Difficulty	-1.93	1.11	-1.68	0.44	-0.02	1.67	0.47	1.07
Lowest Percentage	93	45	90	62	69	24	62	41
Highest Percentage	100	77	100	88	88	82	100	100

Note. a is for Yes or No. *b* is for number of blocks needed.

Table 4: Item difficulty and class percentages of students correct for items on recognising the blocks and number of blocks needed for a 3Dshape.

Part C2 Look at the block under Smiley. If you had some more like this could you make the 3D shape.

Circle Yes or No.

If Yes, write how many blocks you need.

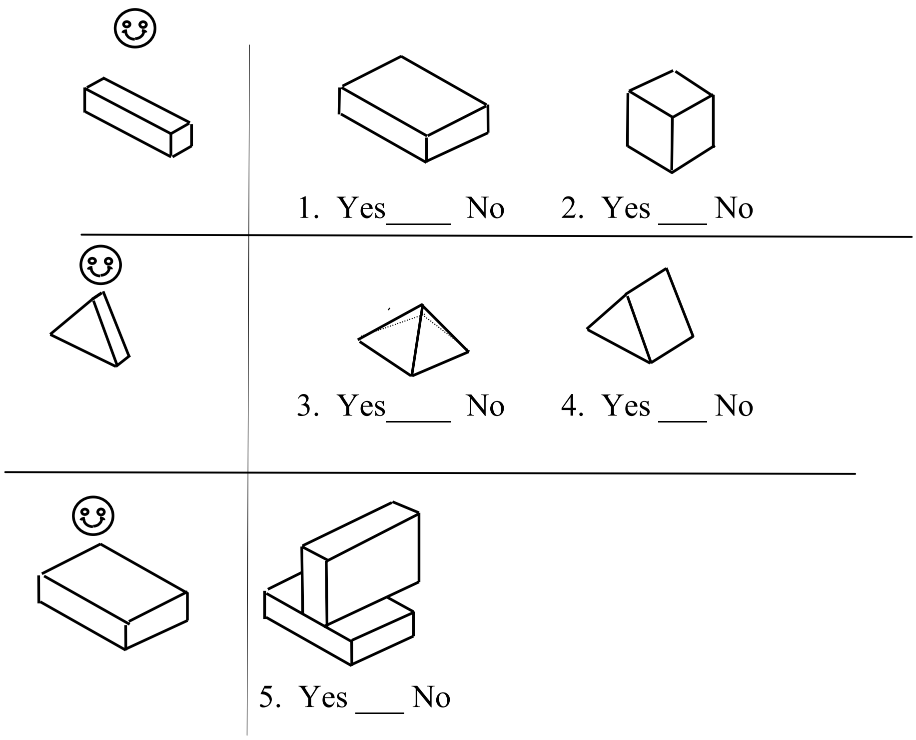

Figure 5: Imagining building with the same blocks.

Ahmed said “Yes” for both C2,3 and C2,4. He reasoned:

Ahmed:	That (points to triangular prism under Smiley) 2 shapes make point (on C2,3). If joined together (claps hands together), it will make that (points to C2,4).
Interviewer:	What was going on in your mind?
Ahmed:	A square
Interviewer:	You seemed to be counting? Can you explain what you were doing?
Ahmed:	It's like a book?
Interviewer:	Can you explain?
Ahmed:	(Counts) one two three four (pointing along the block in C2,4).

On the other hand, Joe said "but [it, the triangle] won't fit into square" (Item C2,4). Joe, like Ahmed, thought of standing triangular blocks up as the faces of the pyramid, not realising that they will not fit together neatly. They may have done this with thin plastic shapes that join together.

In 2D tiling activities, students find it difficult to see how shapes tessellated to give a product that no longer holds a distinct feature like a point. For example, right-angled isosceles triangles make a square but the small pointed angles are no longer obvious (Owens, 1992). For Item C2,4, some students could not see how the square face can be formed from the thin triangular prisms. Some students, especially the younger students (see Table 4), found it difficult to consider size. They are not sure about the size of tiles when covering areas (see, for example, Owens & Outhred, 1998), and two interviewed students counted many rectangular prisms as making up the cube in C2,2. Item C2,5 is interesting as the percentages of students who were correct in some groups were quite low suggesting that experience or interpreting figural representations influences the decision on equality of the blocks.

DISCUSSSION

From this study it is clear that young students from middle primary school are capable of reasoning from pictorial representations of three-dimensional shapes. Nevertheless, their focus of attention when answering a question may prevent students reasoning adequately about the 3D shapes. For example in the building of blocks with the arch (Figure 2; B2) we found that lower achieving students were less able to realise that the shorter and full arch block (B2,2) was not one of the blocks used. It was difficult for students to realise that the half arch (B2,3) was one of the required blocks. More opportunities to play with a variety of blocks would help students to read the diagrams. Students need the opportunity to discuss what the pictures represent. This was apparent during our introduction to the test when we used concrete examples, drew them in isometric representation (like the diagrams above), and discussed various features of the diagrams. These discussions covered a variety of prisms and types of rectangular prisms including those made by joining two rectangular prisms together. Teachers regularly commented on the value for their students of these introductions. It seems they had not had similar discussions with their students. They also found the test itself a valuable experience as students were not often asked to reason about such diagrams. Most diagrams in their 3D lessons were single unit shapes like a picture of a cube or a triangular prism.

We also found that some students changed their minds during the interviews. Students clarified or assessed their current thinking and decisions. This was the case for items B3,3 and B3,4 (Figure 3) on making rectangular prisms. Students were synthesising verbal and imagistic knowledge. They directed their attention to parts of the shapes and considered that the blocks could be manipulated rather than be static representations. Students applied a range of mental spatial strategies including analysis and rotation. Students need the opportunity to talk aloud about their thinking as they did during the introductions and interviews. In the items of C1 (Figure 4), the students noticed the parts of the shapes and talked about how these parts helped them solve the problems. Teachers should ask students to just look at the diagram, look for different things in the same diagram, talk about what they notice and reason from what they notice. These learning strategies are critical experiences for students to develop their abilities to notice and see parts of shapes and to see parts and wholes in different ways.

Students are able to imagine partially hidden shapes and use this information in analysis. They re-see the image in order to notice all the relevant properties. We see this in the discussion of items in B2 (Figure 2). For example, they know what kind of faces and corners exist even though they are not seen. Mason (2003) discusses how past experience and taking time to look hard at a configuration will assist this mental imagery. Both concrete experiences and trying to draw and read diagrams of blocks will facilitate students' spatial reasoning.

This study illustrates the diversity of students' thinking at the initial stages of reasoning about 3D shapes. This diversity suggests that students' development cannot be explained by a simplistic hierarchical structure of noticing one and then many properties. Such an approach may be educationally restrictive if teachers focus on the outcome, the naming of properties and the labelling of the students' development. Teachers need to realise the role of image making and having, disembedding and noticing in students' development and to provide the diversity of visual and manipulative experiences required by students. The levels of difficulty varied considerably for these items illustrating just how some diagrams can be quite difficult for students to perceive parts of shapes and to manipulate and reason from their imagery. Evidence of the diversity of students' noticing and perceiving can be gleaned from the above transcripts of the students' interviews but also from the tables illustrating the item difficulties and percentage correct.

While more students in older age groups tended to perform better on harder items, this was not always the case. Sometimes it was a lower class who performed well on an item. From students' comments about their past experiences and the diversity of achievement from class to class, we can conclude that there is an experiental factor in achievement. If this is the case, then we need to give our students a greater variety of experiences with 3D shapes, manipulating paper to make 3D models, playing and building with different shaped blocks, learning to read a greater variety of diagrams and drawing a greater variety of 3D objects and collections of objects. Students need opportunities to talk about what they see and listen to other students' opinions. In this way, students see, notice and attend to a greater variety of features of 3D shapes and collections of 3D shapes. This is particularly the case where blocks have to be imagined such as stacking together 3D tiles or where one feature of the shape (e.g. the triangular face) dominates over the remaining features of the shape.

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